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Some Interesting Situations in the Hydrodynamics of Smectics and Cholesterics

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SOME INTERESTING SITUATIONS IN THE HYDRODYNAMICS OF SMECTICS AND CHOLESTERICS

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ABSTRACT

We seek solutions to the hydrodynamical equations of smectics and cholesterics in three different circumstances: (a) When permeation is present we find that viscous heating plays a very important part, (b) An arbitrary but constant phase slippage can result in an AC Josephson effect, (c) Thermal expansion of the layers brings down the Bénard threshold markedly.

The hydrodynamics of smectic A can be understood in terms of the continuum model given by de Gennes[1] and later elaborated by Martin et al.[2] The basic equations are:

$$v_{i,i} + \frac{1}{\rho}\frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial v_i}{\partial t}\rho = - P_{,i} + g\delta_{i3} + t'_{ij,j}$$

$$T\dot{S}\rho = d_{ij}\, t'_{ij} + g(\dot{u} - v_z) + K_{\perp}T_{,ii} + (K_{\parallel} - K_{\perp})T_{,33}$$

$$\dot{u} - v_z = \lambda_p g \quad .$$

These equations also describe the hydrodynamics of cholesteric liquid crystals in the long wavelength limit.

In this case $\dot{u}$ can arise not only from displacements parallel to the twist axis but also from rotations about it.

We consider below these equations in situations where the generally accepted approximations are not valid.

1. Heating due to permeation

In situations where permeation is important the structure is firmly anchored or blocked during fluid flow. This permeation flow has been studied by many,[3,4] but it has always been assumed that heating due to viscous motion is negligible. This approximation is reasonably good for classical fluids. But it does not appear to have been stressed that heating effects are very important when permeation is present.

As an example we consider the blocked cholesteric or smectic texture between two parallel planes or in a capillary. Hence $\dot{u} = 0$, and the above system of equations allow a steady viscous flow under an imposed pressure gradient $P_{,z}$ along the Z-axis.

$$P_{,z} = -\frac{v_z}{\lambda_P} + \bar{\eta}\nabla^2 v_z \qquad (1)$$

Here $\bar{\eta}$ is an average viscosity of the order of 10^{-2} poise. The permeation coefficient λ_P is generally of the order of[4] $(\bar{\eta}q_o^2)^{-1}$, with $d = 2\pi/q_o$ as the structural periodicity along the Z-axis. Since d is usually very small compared to the dimensions of the channel one finds a flat velocity profile $v_z = -\lambda_P P_{,z}$ over most of the cross section. Now the entropy equation gives

$$K_{\parallel}\frac{\partial^2 T}{\partial Z^2} + K_{\perp}\nabla_{\perp}^2 T = -(2\bar{\eta})\, d_{ij}^2 - \left(\frac{1}{\lambda_P}\right) v_z^2 \qquad (2)$$

In normal fluids we have only the term in d_{ij} and it is very small. However in smectic A or cholesterics we have the second term on the right hand side of (2) which cannot be ignored. In permeation flow this term becomes very important and to a good approximation the temperature profile is given by

$$K_{\perp} \nabla_{\perp}^2 T = - \frac{v_z^2}{\lambda_P} \quad . \qquad (3)$$

For capillary flow this yields

$$T(r) - T(R) = \frac{1}{4} v_z^2 \left(\frac{\bar{\eta}}{K_{\perp}}\right) \left(1 - \frac{r^2}{R^2}\right) (q_o R)^2 \quad . \qquad (4)$$

On the other hand for normal fluids in the same geometry[5]

$$T(r) - T(R) = \frac{1}{2} v_m^2 \left(\frac{\eta}{K}\right) \left(1 - \frac{r^4}{R^4}\right) \qquad (5)$$

R being the capillary radius and v_m the mean flow velocity.

A comparison of (4) and (5) shows that for the same flow velocity the heating effects in the permeation mode could be very large, about a million times, through the factor $(q_o R)^2$. Also the profiles are different in the two cases.

2) Phase slippage: AC Josephson Effect

We now assume $\dot{u}$ *not* to be zero, but a constant, $\tau\lambda_P$ say. This quantity represents the rate at which the phase of a layer is changing, i.e., phase slippage. How this can be effected will be considered later. In such a situation, in a capillary flow under an imposed pressure gradient

$$v_z = - \lambda_P (P_{,z} - \tau) \left(1 - \frac{I_o(kr)}{I_o(kR)}\right)$$

I_o being the Bessel function of first kind in zero order and $k^2 = 1/\bar{\eta}\lambda_p$. In general, phase slippage results in an extra pressure gradient, and for $P_{,z} = \tau$ the flow velocity v_z vanishes completely. This is akin to the AC Josephson effect in superfluid Helium.[6,7] Here a column of superfluid Helium II is allowed to flow out of a narrow orifice into a lower tank. A tuning fork of adjustable frequency is kept at the orifice to generate vortices. Experimentally it is found that the liquid column can be held stationary against gravity only at particular frequencies.

We shall now consider the problem of effecting phase slippage in smectics or cholesterics. Being layered structures, they allow edge dislocations to exist in them with the dislocation line in the plane of the layers. On going from one side of the dislocation line to the other on any given layer, u changes by d the lattice parameter or, in effect, the phase changes by 2π (as in the case of a vortex). Hence a migration of dislocations at a constant rate γ gives the required $\dot{u}$. This is exactly the mechanism proposed by Anderson[6] to explain the AC Josephson effect in superfluid Helium II.

In the case of cholesterics, in addition to the above process, we can get the required $\dot{u}$ by turning the system rigidly above the twist axis at a constant rate γ, all this happening near the orifice.

3) Thermoelastic Effects

Both smectic A and cholesterics have a lattice structure along the Z-axis. We know from experiments that the lattice parameter is very sensitive to temperature, the thermal expansion coefficient α being one or two orders greater than that in crystals. The large thermal expansion leads to some interesting consequences that do not appear to have been studied.

Thermal expansion of the lattice alters the equations of motion in two ways. Firstly the force g due to the elastic deformation of the structure gets an additional term $\alpha B T_{,z}$, and secondly the rate of entropy generation has an additional term $\alpha B \frac{\partial \dot{u}}{\partial z}$. Here B is the elastic constant for lattice stretching (see references in 8 for introducing corrections due to α). Two immediate consequences of these corrections are -

i) Thermo-elastic coupling due to α simulates the pure thermo-mechanical effects in cholesterics[9,10] and therefore must not be ignored while discussing pure thermo-mechanical effects.

ii) Temperature fluctuations should accompany phase fluctuations in second sound with an amplitude

$$f = \frac{T\alpha}{\rho C} u'$$

u' is the amplitude of the strain in the phase wave and C the specific heat. Also this results in an extra damping in the second sound through the finite thermal conductivity.

While investigating Bénard instabilities, in cholesterics Parsons[11] and later Pleiner and Brand[12] found that under the assumption of low thermal expansion, the correction to the threshold gradient is negligibly small. In particular they overlooked one possibility. We know that raising (or lowering) the temperature of a smectic confined between plates can result in a Helfrich-Hurault instabibility[13] through the ensuing lattice strain ε_o. (In this sense it is analogous to a destabilizing magnetic field.) The free energy density due to lattice expansion under these circumstances is exactly like that due to a mechanical lattice strain ε_o[14,15] :

$$F = \frac{B}{2}\varepsilon_o^2 + \frac{1}{2} K_{11}\left(\frac{\partial^2 u}{\partial x^2}\right)^2 - \frac{B\varepsilon_o}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{B}{2}\left(\frac{\partial u}{\partial z}\right)^2 \quad .$$

This gives an undulation instability at $\varepsilon_o^c = \frac{2}{D}(K_{11}/B)^{\frac{1}{2}}\pi$, D being the sample thickness. This process, like the destabilising magnetic field[16] brings down the Bénard threshold β_o to β given by

$$\beta = \beta_o\left(1 - \frac{\varepsilon_o}{\varepsilon_o^c}\right)$$

This could be an important process in view of the fact that $\varepsilon_o^c \sim 10^{-4}$ and strains of this magnitude are attainable through lattice expansion.

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